Int. J. Heat Mass Transfer. Vol. 32, No. 4, pp. 761–765, 1989
Printed in Great Britain

0017–9310/89 \$3.00+0.00

Turbulent boundary layer on a porous flat plate with severe injection at various angles to the surface

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(*Received* 15 *September* 1987 *and in final form* 9 *June* 1988)

Abstract—The paper describes the calculation of a boundry layer on a permeable flat plate with severe injection at different rates and at different angles to the surface. Two turbulent models are employed, namely, the k–W and k–ε models. Emphasis is placed on the treatment of the near-wall region to take account of the magnitude and direction of injection. Predictions based on these features are compared with the data of Yeroshenko *et al.* (*Heat Transfer—Sov. Res.* **13**(5), 13 (1981)) for the mean velocity and turbulence fluctuations. The calculations also go beyond the experimental investigation by providing the distribution of shear stresses.

1. INTRODUCTION

INJECTION of secondary fluid through porous walls is of practical importance in film cooling of turbine blades and combustion chambers. In such applications, injection usually occurs normal to the surface and the injected fluid may be similar to or different from the primary fluid. In some recent applications however, it has been recognized that cooling efficiency can be enhanced by vectored injection at angles other than 90° to the surface. Indeed, a few workers including Inger and Swearn [1] have theoretically proved this feature for a linear boundary layer.

While a great deal of work such as in refs. [2, 3] has been carried out on normal injection, the problem of vectored injection has received comparatively less attention. In addition, most previous calculations have been limited to injection rates ranging from small to moderate. This trend is partly due to the complexity of these flow situations. Furthermore, success in modelling them is still modest because of possible 'elliptic' effects, i.e. normal-to-wall pressure gradients, for reverse injection on the one hand, and inadequacy of turbulence models which neglect pressure gradient effects on the other. Nevertheless, there is still the need to explore the possibility of using existing calculation models to predict characteristics of practical interest for such complex flows.

The present work is an attempt in that direction. A boundary layer with severe vectored injection is calculated with a parabolic solution procedure and two turbulence models namely, k–W [4] and k–ε models [5] in which k is the specific turbulence energy, W the time-mean square vorticity fluctuations and ε the rate of turbulence energy dissipation. Modifications are made to the smooth wall functions to account for the effect of mass transfer and inclination of injection to the surface. The calculations are then compared with the experimental data of Yeroshenko *et al.* [6] where available. The experimental study concerns an isothermal system at flow velocities sufficient to produce a developed turbulent boundary layer. The primary working fluid is nitrogen while the secondary fluids are separately nitrogen and carbon dioxide. The calculations reported here however consider only the injection of nitrogen.

The performances of the two turbulence models employed are compared with each other. The predictions are also carried beyond the experimental investigation by providing the distribution of shear stresses across the boundary layer. In the following section, the mathematical framework is developed. A brief description of the solution procedure is given in Section 3 while the results are presented in Section 4 and discussed in Section 5. Section 6 contains the conclusion.

2. MATHEMATICAL FORMULATION

The equation governing the transport of a generic property ϕ in a steady turbulent flow can be expressed as

$$\frac{\partial}{\partial x_i}(\rho u_i \phi) = \frac{\partial}{\partial x_i}\left(\Gamma_{\text{eff}} \frac{\partial \phi}{\partial x_i}\right) + S_\phi \qquad (1)$$

where ϕ represents u, k, ε, W or 1 (for the mass conservation equation), S_ϕ the source term appropriate to each transport equation and Γ_{eff} the effective exchange coefficient which comprises of molecular and eddy components. Further details of this aspect of the mathematical model are available in refs. [4, 5]. It should however be stressed that the equations are subject to the usual boundary-layer approximations.

2.1. *The near-wall model*

When mass transfer occurs across the solid surface, the shear stress varies significantly between the surface

NOMENCLATURE

E	sublayer parameter
F	mass transfer per unit surface area
k	turbulent kinetic energy
k'	dimensionless turbulence intensity
S_ϕ	source term in the transport equation for ϕ
u	velocity
u_G	freestream velocity
u'	turbulence intensity
$u'v'$	turbulent shear stress
W	root-mean-square vorticity fluctuations
x_i	generalized coordinate
y_p	perpendicular distance of near-wall node from wall
y^+	Reynolds number based on y_p and shear velocity.

Greek symbols

α	angle of inclination of injection to the surface
Γ_{eff}	effective exchange coefficient
δ	boundary-layer thickness
ε	rate of dissipation of turbulence energy
κ	von Karman constant
μ	viscosity
ρ	density
σ	Prandtl number
τ	shear stress
ϕ	generic flow variable
Ω	major component of vorticity.

and the near-wall grid point. There exists no universal relationship to express this feature.

Spalding [7] proposed a method in which an average value of stress is assumed to exist within a couette flow region close to the wall ($y^+ < 11.5$) such that

$$\tau = \tau_w + \tau_m \quad (2)$$

where τ_w is the value of the wall shear stress in the absence of mass transfer and τ_m the contribution due to mass transfer.

This proposal has been applied successfully to flows with normal injection in ref. [4]. However, in order to extend it to the present situation, the inclination of the injection needs to be considered.

Generally, the injection rate $F = F(F_x, F_y)$ where

$$F_x = F \cos \alpha \quad (3)$$

$$F_y = F \sin \alpha \quad (4)$$

and α is the inclination of injection to the surface.

The influence of F_y which will be given shortly, is treated exactly as suggested in ref. [7]. F_x is here assumed to impart a velocity Δu of the order of F_x/ρ on the fluid in the grid cell adjacent to the wall such that the effective velocity in that node is $u - \Delta u$. Therefore, following Spalding [7] and introducing the above approximation, a drag law similar to that for the smooth wall is employed in the form

$$\frac{u-\Delta u}{(\tau_w/\rho)^{1/2}} = \frac{1}{\kappa} \ln\left[\frac{E y_T (\tau\rho)^{1/2}}{\mu}\right] \quad (5)$$

and

$$\tau_m = \frac{F u \sin \alpha}{1 + (\tau_w/\rho)^{1/2}/\kappa u}. \quad (6)$$

In effect, equations (2) and (6) are used to calculate the 'average shear stress' τ which is used on the right-hand side of equation (5). The above formulation of course reduces to that originally proposed for normal injection in ref. [7] when $\alpha = 90°$. Preliminary work indicates that the influence of F_x may be neglected for $|F_x|/\rho u_G \ll 1$.

Assuming the above log-law of the wall and local equilibrium in the couette flow region leads to the following relation between the shear stress and turbulent energy k_p at the near-wall grid node:

$$k_p = (c_\mu c_d)^{1/2} (\tau/\rho) \quad (7)$$

where $c_\mu = 1.0$ and $c_d = 0.09$.

Finally, assuming that the length scale is a linear function of the distance y_p from the wall gives the following boundary conditions on W and ε:

$$W_p = \kappa^{-2} c_d^{-1/2} k / y_p^2 \quad (8)$$

and

$$\varepsilon_p = \kappa^{-1} c_d^{3/4} k^{3/2} / y_p. \quad (9)$$

3. SOLUTION OF THE EQUATIONS

The differential equations for k, W, ε, together with those governing the velocity components are solved by a forward-matching, finite-domain, fully-implicit solution procedure embodied in the PHOENICS computer code [8].

The grid width is allowed to expand with downstream distance such that it covers only regions where flow properties vary. Grid independence is achieved with 48 non-uniformly distributed lateral grid cells and a forward step size of 2% of the local width of the shear layer.

4. RESULTS

Figure 1 shows profiles of the predicted mean velocities for an injection angle (α) of 15° for three injec-

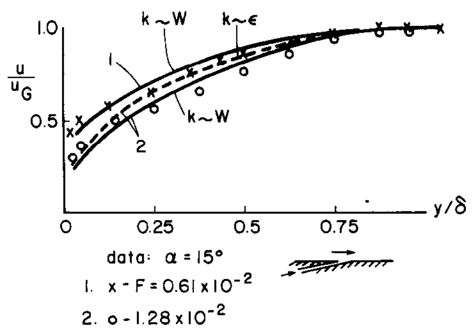

FIG. 1. Profiles of velocity for injection at 15°.

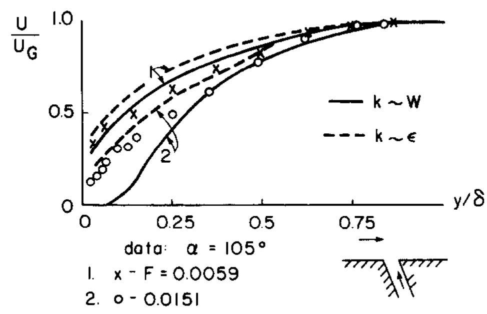

FIG. 3. Profiles of velocity for injection at 105°.

tion rates. Also shown are the data reported in ref. [6]. Both models are seen to perform satisfactorily at this low injection angle.

Corresponding sets of results for $\alpha = 90°$ and $105°$ are shown respectively in Figs. 2 and 3. The k–W model generally underpredicts the velocity in the near-wall region especially at the higher injection rate, while the k–ε model overpredicts the same for both injection rates considered.

In Figs. 4–6 are shown respectively the profiles of the turbulence intensity at $\alpha = 15°$, $90°$ and $105°$. Once again, the agreement for $\alpha = 15°$ is quite satisfactory across the boundary layer. It is seen however that at the higher injection angles, significant discrepancies exist between measurements and predictions especially at the higher injection rate. In all cases, the maximum value of the intensity increases with the injection rate.

Figures 7–9 show predicted profiles of turbulent shear stress at $\alpha = 15°$, $90°$ and $105°$, respectively. Unfortunately, there are no reported experimental data for direct comparison. It is interesting however, that while the maximum stress increases with injection rate for $\alpha < 90°$, the reverse seems to be the case at the higher injection angle, $\alpha > 90°$. In all cases, the turbulence shear stress profiles in the outer region appear to be essentially independent of the injection rate and injection angle. In addition, a comparison of these figures shows that the maximum shear stress decreases as the injection angle increases.

5. DISCUSSION

The above results compare on the one hand, the performance of the k–W model with the k–ε model, and the performance of both models with the measurements on the other. The principal findings may be summarized as follows.

(a) The performances of both models are generally comparable.

(b) Results for downstream-directed injection are essentially similar to those for normal injection while significant qualitative differences exist for upstream-directed ones.

(c) The agreement between predictions and measurements may be considered generally satisfactory at low injection angles and low injection rates, but deteriorates at the higher values. This trend may be attributed to the strong ellipticity at the higher injection angles; a feature that is not accounted for in the calculation scheme. Besides, the k–W and k–ε models have no means of representing this effect. In addition, it is doubtful whether the approximate treatment of the near-wall region using an average shear stress is valid at high injection rates.

(d) The intensity of turbulence increases with injection rate due to the increasingly strong influence on the main flow. The maximum value of the intensity also appears to increase with injection angle especially for $\alpha > 90°$. This behavior is consistent with the fact

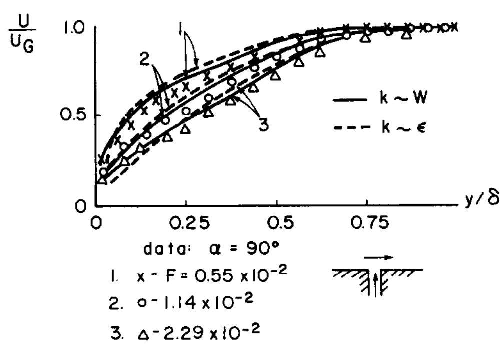

FIG. 2. Profiles of velocity for injection at 90°.

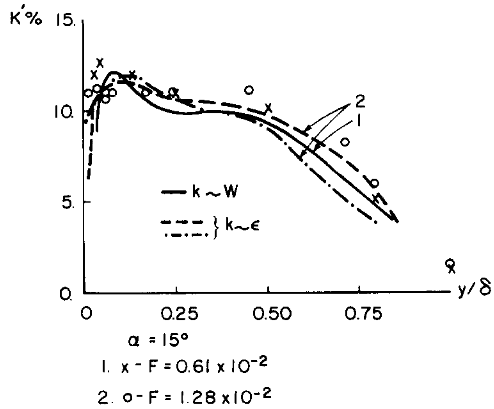

FIG. 4. Profiles of turbulence intensity at 15°.

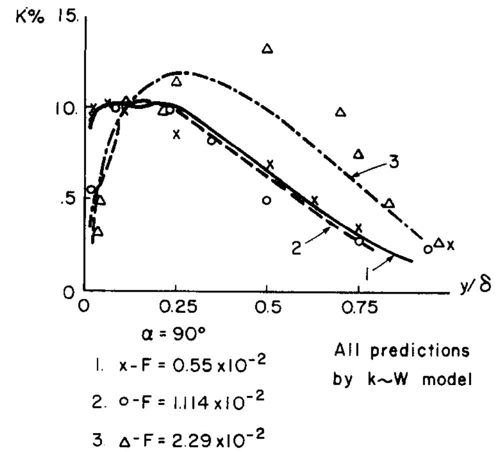

FIG. 5(a). Profiles of turbulence intensity at 90° including k–W predictions.

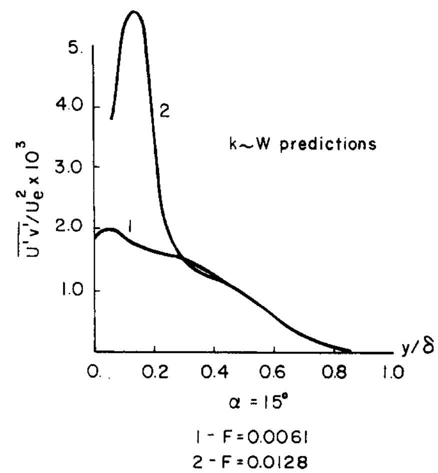

FIG. 7. Predicted turbulent shear stresses for injection at 15°.

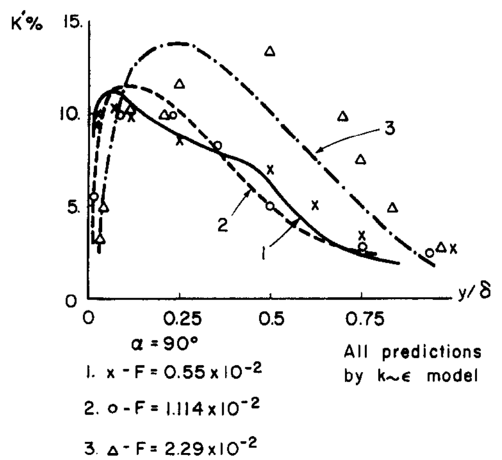

FIG. 5(b). Profiles of turbulence intensity at 90° including k–ε predictions.

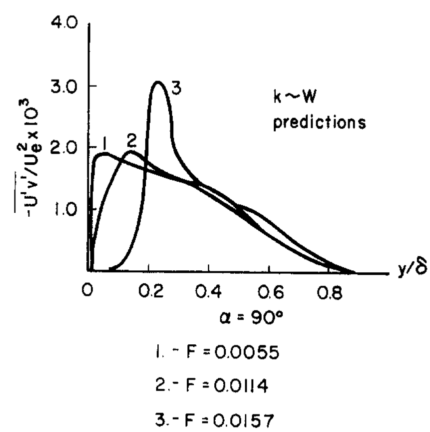

FIG. 8. Predicted turbulent shear stresses for injection at 90°.

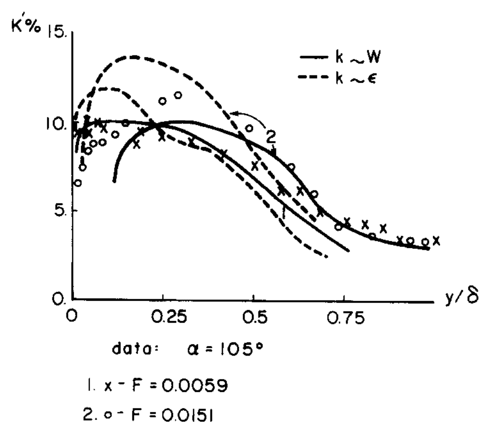

FIG. 6. Profiles of turbulence intensity at 105°.

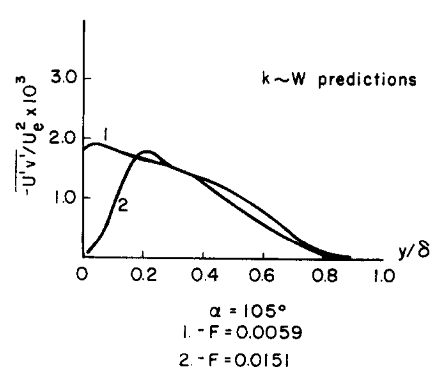

FIG. 9. Predicted turbulent shear stresses for injection at 105°.

that an increase in α for the latter case increases the boundary-layer thickness, and also enhances redistribution of the turbulence intensity within the boundary layer.

(e) At approximately the same injection rate, the maximum shear stress decreases as the injection angle increases. However, there appears to be no apparent correlation between this maximum value and the injection angle.

6. CONCLUSION

Calculations have been presented of the velocity and turbulence characteristics of a boundary layer into which a fluid similar to the primary flow is injected at various rates and angles to the surface. In deriving wall functions, an averge shear stress is used, that accounts for both the magnitude and direction of injection.

The results have been compared with the experimental data where available. The qualitative agreement between predictions and measurement is generally satisfactory, and there is little difference between the performances of the k–W and k–ε models.

These results have shown that two-equation models, coupled with slight modifications to the wall functions, can still predict satisfactorily the essential characteristics of such complex boundary-layer flows.

Acknowledgements—The author would like to thank Professor D. B. Spalding and Dr S. Zhubrin of Moscow Technical Institute for suggesting this problem and for helpful discussion.

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COUCHE LIMITE TURBULENTE SUR UNE PLAQUE PLANE POREUSE AVEC UNE INJECTION SEVERE A DIVERS ANGLES AVEC LA SURFACE

Résumé—On décrit le calcul de la couche limite sur une plaque plane perméable avec une sévère injection à différentes vitesses et divers angles avec la surface. Deux modèles de turbulence sont utilisés; les modèles k–W et k–ε. Une attention particulière est portée sur le traitement de la région proche de la paroi pour tenir compte de l'intensité et de la direction de l'injection. Des prédictions basées sur ces idées sont comparées avec les données de Yeroshenko *et alii* (*Heat Transfer—Sov. Res.* **13**(5), 13 (1981)) pour la vitesse moyenne et les fluctuations turbulentes. Les calculs vont aussi au delà de l'etude expérimentale en fournissant la distribution des contraintes tangentielles.

DIE TURBULENTE GRENZSCHICHT AN EINER PORÖSEN EBENEN PLATTE MIT MEHRFACHER EINSPRITZUNG UNTER UNTERSCHIEDLICHEM WINKEL ZUR OBERFLÄCHE

Zusammenfassung—Der Beitrag beschreibt die Berechnung einer Grenzschicht an einer durchlässigen ebenen Platte mit mehrfacher Einspritzung mit unterschiedlichem Massenstrom und unterschiedlichem Winkel zur Oberfläche. Es werden zwei Turbulenz-Modelle angewandt, das k–W-Modell und das k–ε-Modell. Besondere Sorgfalt wird auf das wandnahe Gebiet verwendet, um Menge und Richtung der Einspritzung zu berücksichtigen. Die so durchgeführten Berechnungen werden mit experimentellen Ergebnissen für die mittlere Geschwindigkeit und Turbulenzschwankungen von Yeroshenko *et al.* (*Heat Transfer—Sov. Res.* **13**(5), 13(1981)) verglichen. Die Berechnungen gehen über die experimentellen Untersuchungen hinaus, indem sie die Verteilung der Schubspannungen liefern.

ТУРБУЛЕНТНЫЙ ПОГРАНИЧНЫЙ СЛОЙ НА ПОРИСТОЙ ПЛОСКОЙ ПЛАСТИНЕ ПРИ СИЛЬНОМ ВДУВЕ ПОД РАЗЛИЧНЫМИ УГЛАМИ К ПОВЕРХНОСТИ

Аннотация—Описан расчет пограничного слоя на проницаемой плоской пластине при сильном вдуве различной интенсивности и под различными углами к поверхности. Используются две модели турбулентности, а именно, модели k–W и k–ε. Особое внимание обращается на изучение пристенной области для учета величины и направления вдува. Основанные на этих характеристиках определения сравниваются с данными, полученными Ерошенко и др. (*Heat Transfer—Sov. Res.* **13**(5), 13 (1981)) для средней скорости и турбулентных пульсаций. Рассчитано также распределение напряжения сдвига.